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SIMULATION METHODOLOGY

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and

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FINAL REPORT

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l. Introduction

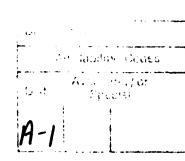
This contract covered the period March 1, 1984 to December 31, 1987. The participating scientific personnel included two faculty members and seven Ph.D. students. Three of the students completed both an M.S. in Statistics and a Ph.D. in Operations Research, one of whom also earned an M.S. in Operations Research. The other four students are currently Ph.D. candidates in Operations Research, one of whom has earned an M.S. in Operations Research and an M.S. in Statistics.

The main thrust of our research has been in developing new methods for analyzing the output from computer simulations. The technical work can be divided into three areas: simulation output analysis, variance reduction, and stochastic optimization. Below we mention some of the most important results in each area.

2. Simulation Output Analysis

Glynn and Iglehart (Technical Report No. 8) developed theoretical foundations for the so-called standardized time series approach introduced by Schruben. This method addresses the problem of estimating a steady-state parameter for a process satisfying a functional central limit theorem





(CLT). In this approach, rather than estimating the variance constant appearing in the CLT, this constant is cancelled out in a manner reminiscent of the t-statistic. The method has a number of interesting variants depending on the selection of a certain function. One of these selections leads to the method of batch means with a fixed number of batches.

Birney Titus in his Ph.D. Dissertation (Technical Report No. 9) studied the problem of estimating the mean of a stationary autoregressive (AR) process of fixed order, p. In previous work by Jow and others, it has been shown that simulation output from an asymptotically stationary process can be approximated by an AR(p) process. Titus developed a method using Edgeworth expansions for constructing modified confidence intervals which have better coverage characteristics than those previously available.

Glynn and Iglehart (1986) considered the problem of estimating the steady-state central moments for a positive recurrent regenerative process. In this paper strong laws, central limit theorems and confidence intervals were obtained and illustrative numerical examples presented.

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Haas and Shedler (1986) studied regenerative stochastic Petri nets. Petri nets constitute a modeling formalism that is different from that traditionally used in stochastic models arising in operations research. They are particularly useful in modeling a variety of local area computer networks. Haas and Shedler developed methods for constructing confidence intervals for steady-state parameters.

Glynn and Iglehart (1987) obtained a joint central limit theorem for the sample mean and the regenerative variance estimator in the context of regenerative simulation. This result has led to a number of interesting consequences. It was known previously that the variance associated with the sample mean is independent of the return state used. However, from this CLT it became apparent thru numerical examples that the covariance term associated with the sample mean and the variance estimator should also be independent of the return state used. This has subsequently been proved by Jim Calvin and will appear in a future technical report. Also, this work has shown that the variance of the variance estimator is not minimized (necessarily) by the state having the smallest expected cycle length (in contrast to the 'folklore' of this area); a better heuristic is to choose a frequently visited state in the 'center' of the space, so as to minimize skewness/asymmetry effects. Finally, numerical examples have shown that the variance of the variance estimator can be very large.

Glynn (1988) developed a new low-bias steady-state estimator for equilibrium processes. Steady-state estimators, such as the ratio estimators which arise in regenerative simulation, are normally biased (although asymptotically unbiased). Glynn has computed the first-order bias term which can be estimated from the simulation data. The estimate for this bias term can then be subtracted from the classical estimate to obtain the low-bias estimate. Numerical examples have shown that this estimator is very effective in reducing bias.

3. Variance Reduction Techniques

The method of control variables generally assumes that the expected values of the control variables are known exactly. Glynn and Iglehart (Technical Report No. 17) considered the case where these expected values are only known asymptotically. In this paper the optimal linear combination

of controls is derived. This analysis is particularly relevant to the small-sample theory for control variates as applied to the steady-state estimation problem.

Fox and Glynn (Technical Report No. 23) studied the problem of estimating the expected infinite-horizon discounted cost d of running a stochastic system. By exploiting various types of stochastic structure they are able to construct estimators that exhibit smaller variances than the naive strategy of estimating a finite-horizon approximation to d.

Importance sampling is one of the classical variance reduction techniques that has been used in the evaluation of integrals. Glynn and Iglehart (Technical Report No. 24) have used importance sampling in the context of discrete and continuous time Markov chains and generalized semi-Markov processes. This method is also shown to lead to variance reduction.

4. Stochastic Optimization

Glynn (Technical Report No. 14) has studied the case of a discrete-time Markov chain whose transition matrix depends on a parameter θ . Let $\alpha(\theta)$ denote the steady-state cost of running this system per unit time. Numerical and simulation methods are derived for calculating and estimating the gradient of $\alpha(\theta)$. These gradient estimates have also been used to develop convergent simulation algorithms for optimizing complex stochastic systems in which the decision parameters are continuous.

The likelihood ratio method has been developed recently in order to estimate the gradient of the cost of running a stochastic system that

depends on a parameter θ . Glynn (Technical Report No. 26) surveys this method and its application to transient and steady-state problems. For the steady-state problem both regenerative and non-regenerative approaches are discussed.

5. Scientific Personnel

The following scientific personnel were supported on this contract:

Faculty supported:

Donald L. Iglehart, Principal Investigator Peter W. Glynn, Co-investigator

Students supported:

Birney D. Titus, earned M.S. in Statistics and Ph.D. in Operations Research.

Peter J. Haas, earned M.S. in Statistics and Ph.D. in Operations Research.

Lindsay Prisgrove, earned M.S. in Statistics and Ph.D. in Operations Research.

James Calvin, Ph.D. candidate in Operations Research.

Chia-Hon Chien, earned M.S. in Statistics and M.S. in Operations Research, Ph.D. candidate in Operations Research.

Nayyar P. Shahabuddin, Ph.D. candidate in Operations Research.

Scott D. Schulz, Ph.D. candidate in Operations Research.

6. ARO TECHNICAL REPORTS

Number	Date	Title	Author
1	6/84	Regenerative Simulation Methods for Local Area Computer Networks	Peter J. Haas Gerald S. Shedler
2	9/84	Regenerative Simulation of Non-Markovian Stochastic Systems	Donald L. Iglehart Gerald S. Shedler
3	9/84	Confidence Intervals Using the Regenerative Method for Simulation Output Analysis	Peter W. Glynn Donald L. Iglehart
4	10/84	Regenerative Stochastic Petri Nets	Peter J. Haas Gerald S. Shedler
5	12/84	Recursive Moment Formulas for Regenerative Simulation	Peter W. Glynn Donald L. Iglehart
6	3/85	A Joint Central Limit Theorem for the Sample Mean and Regenerative Variance Estimator	Peter W. Glynn Donald L. Iglehart
7	3/85	Symmetric Stochastic Petri Nets	Lindsay A. Prisgrove Gerald S. Shedler
8	4/85	The Theory of Standardized Time Series	Peter W. Glynn Donald L. Iglehart
9	8/85	Modified Confidence Intervals for the Mean of an Autoregressive Process	Birney D. Titus
10	8/85	Large-Sample Theory for Standardized Time Series: An Overview	Peter W. Glynn Donald L. Iglehart
11	8/85	Estimation of Steady-State Central Moments by the Regenerative Method of Simulation	Peter W. Glynn Donald L. Iglehart
12	3/86	The Optimal Linear Combination of Control Variates in the Presence of Bias	Peter W. Glynn Donald L. Iglehart
13	4/86	Recursive Moment Formulas for Semi-Markov Processes	Chia-Hon Chien
14	8/86	Sensitivity Analysis for Stationary Probabilities of a Markov Chain	Peter W. Glynn
15	10/86	Consequences of Uniform Integrability for Simulation	Peter W. Glynn Donald L. Iglehart

16	10/86	A New Class of Strongly Consistent Variance Estimators for Steady-State Simulations	Peter W. Glynn Donald L. Iglehart
21	4/87	Conditions Under Which a Markov Chain Converges to its Steady-State in Finite Time	Peter W. Glynn Donald L. Iglehart
22	4/87	A Low Bias Steady-State Estimator for Equilibrium Processes	Peter W. Glynn
23	8/87	Simulating Discounted Costs	Bennett L. Fox Peter W. Glynn
24	8/87	Importance Sampling for Stochastic Simulations	Peter W. Glynn Donald L. Iglehart
25	9/87	A New Initial Bias Deletion Rule	Peter W. Glynn Donald L. Iglehart
26	9/87	Likelihood Ratio Gradient Estimation: An Overview	Peter W. Glynn

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- Glynn, P.W. and Iglehart, D.L., Confidence Intervals Using the Regenerative Method for Simulation Output Analysis, Proceedings of the 1984 Winter Simulation Conference, Dallas, TX (1984), 248-249.
- Glynn, P.W. and Iglehart, D.L., Large-sample Theory for Standardized Time Series: An Overview, <u>Proceedings of the 1985 Winter Simulation Conference</u>, San Francisco, CA (1985), 129-134.
- Glynn, P.W. and Iglehart, D.L., Recursive Moment Formulas for Regenerative Simulation, Semi-Markov Models [J. Jannsen, ed.] (1986), 99-110, Plenum Press, New York.
- Glynn, P.W. and Iglehart, D.L., Estimation of Steady-State Central Moments by the Regenerative Method of Simulation, Operations Research Letters 5 (1986), 271-276.
- Glynn, P.W. and Iglehart, D.L., A Joint Central Limit Theorem for the Sample Mean and Regenerative Variance Estimator, Annals of Operations Research 8 (1987), 41-55.
- Glynn, P.W., Sensitivity Analysis for Stationary Probabilities of a Markov Chain, Proceedings of the 4th Army Conference on Applied Mathematics and Computing.
- Glynn, P.W. and Iglehart, D.L., The Optimal Linear Combination of Control Variables in the Presence of Bias, Naval Research Logistics Quarterly, to appear.
- Glynn, P.W. and Iglehart, D.L., The Theory of Standardized Time Series, Mathematics of Operations Research, to appear.
- Glynn, P.W. and Iglehart, D.L., A New Class of Strongly Consistent Estimators for Steady-State Simulations, Stochastic Processes and their Applications, to appear.
- Glynn, P.W., Low Bias Steady-State Estimator for Equilibrium Processes, Mathematics of Operations Research, to appear.
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Iglehart, D.L. and Shedler, G.S., Simulation for Passage Times in Non-Markovian Networks of Queues, <u>Proceedings of the IFIP Workshop on Stochastic Programming</u>: <u>Algorithms and Applications</u> (1983).

Prisgrove, L. and Shedler, G.S., Symmetric Stochastic Petri Nets, <u>IBM J. of</u> Research and <u>Development</u> 30 (1986), 278-293.

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